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Report on the doctoral thesis 'Spin-polarized impurities in ultracold Fermi gas' by M.Sc. Eng. Buğra Tüzemen

The doctoral thesis by Buğra Tüzemen describes his theoretical investigations of unique features appearing in the strongly-paired superfluid fermion system, upon temporarily imposing the Gaussian spin-resolved potential. Such local (in space and time) disruption of the superfluid state gives rise to emergence of stable *ferron*-type structure, manifested in the local magnetic polarization and very specific inhomogeneity of the pairing field. The latter one is characterized by: (a) concentric region where the pairing is substantialy suppressed and (b) π-shift of the complex order parameter upon traversing this *nodal* region. The author analyzed in great detail, both the static and dynamic properties of ferrons by means of the density functional approach and using the microscopic Bogoliubov de Gennes formalism. He obtained a number of original and valuable results. Such theoretical predictions could be verified empirically, for instance, in the ultracold fermion atom superfluids at the unitary limit.

The Ph.D. thesis has been prepared in the Nuclear Theory Group in Faculty of Physics at Warsaw University of Technology, under supervision of professor Piotr Magierski. Sophisticated numerical calculations have been obtained also in cooperation with Dr. hab. Gabriel Wlazłowski. These studies have been financed by the Polish National Science Centre, within the research project 2016/23/B/ST2/01789. Major results, constituting the present Ph.D. thesis, have been already published in the following papers:

i) *Spin-polarized droplets in the unitary Fermi gas*
P. Magierski, B. Tüzemen, & G. Wlazłowski,
Phys. Rev. A **100**, 033613 (2019);

ii) *Properties of spin-polarized impurities – ferrons, in the unitary Fermi gas*
B. Tüzemen, P. Kukliński, P. Magierski, & G. Wlazłowski,
Acta Phys. Polon. B **51**, 595 (2020);

iii) *Dynamics of spin-polarized impurity in ultracold Fermi gas*
P. Magierski, B. Tüzemen, & G. Wlazłowski,
Phys. Rev. A **104**, 033304 (2021).

The doctoral thesis consists of 7 chapters. The first part (chapters 1-3) outlines a broad background on superconductivity/superfluidy and presents useful methodological details for modeling the ferron-type objects. In the second part (chapters 4-6) the author presents his original results concerning the static and dynamic properties of ferrons. Chapter 7 summarizes the main findings and provides a brief outlook for closely related future studies. In what follows, I comment on these sections presented by B. Tüzemen in his doctoral thesis.

The introductory chapter 1 refers to the discovery of low temperature superconductivity, which is evidenced by a perfect conductance (dissipationless charge flow) and ideal diamagnetism (total screening of d.c. external magnetic fields). B. Tüzemen confronted this superconducting state of electron system (obeying the Fermi-Dirac statistics) with superfluidity discovered two decades later in the charge-neutral bosonic ^{4}He liquid. Both these spectacular phenomena originate from a long-range coherent behavior of the Bose-Einstein condensed (BEC) ^{4}He atoms or the electron pairs. The author mentioned several pairing scenarios, in particular the resonant Feshbach-type mechanism that is used in the ultracold fermion-atom superfluids. He next addressed the important issue of smooth *crossover* from the superconductivity/superfluidity of the weakly coupled Cooper pairs to the Bose-Einstein condensate of tightly bound fermion pairs. This topic goes back to the early studies by David Eagles and attracted a lot of attention due to discovery of the high T_c superconductivity in cuprates where fermion pairs have the local (inter-site) character. Discrepancy between the fermion pairing and onset of the long-range coherence has been emphasized by such prominent physicists, like: A.J. Leggett, S. Robaszkiewicz with J. Ranninger and R. Micnas, Ph. Nozières with S. Schmitt-Rink, M. Randeria with coworkers, and the Supervisor of this thesis as well. Ultracold fermion superfluids are very appealing in this regard, because their pairing strength (which can be formally expressed in terms of the scattering length) can be varied between these BCS and BEC limits in a controlled manner by virtue of the Feshbach mechanism. The author pointed out several exotic realizations of the superfluid/superconducting state of spin-imbalanced systems ($\mu_\uparrow \neq \mu_\downarrow$), mass-imbalanced case ($m_\uparrow \neq m_\downarrow$), and finite-momentum pairing with the spatially modulated order parameter (FFLO phase). Since these phenomena can be realized in magneto-optically trapped atomic gases at extremely low (submicrokelvin) temperatures, B. Tüzemen presented available empirical tools allowing to reach such necessary quantum degeneracy limit.

Chapter 2 overviews the microscopic theory of conventional superconductivity formulated by Bardeen, Cooper and Schrieffer (BCS model). In particular, the author addressed instability of the Fermi liquid with respect to the Cooper pair formation driven by an effective attraction between electrons. He examined the BCS ground state configuration of the system, comprising a macroscopic number ($N \sim 10^{23}$) of paired electrons and presented derivation of the BCS equations from the Hartree-Fock-Bogoliubov decoupling scheme of 2-body attractive interaction term. He notified, that the resulting Bogoliubov quasiparticles should be represented by a coherent superposition of the particle and hole contributions. This fact was used to express the spatially dependent electron concentration (2.55), kinetic energy (2.56), order parameter (2.57) and charge current (2.58). Next, the author applied the Bogoliubov de Gennes approach to analyze the in-gap bound states formed on interface between the isotropic superconductor and the normal conductor. Such quasiparticles stem from the superconducting proximity effect, i.e. leakage of the Cooper pairs into the normal region on distances comparable to the coherence length ξ. Their presence can be detected through the subgap tunneling spectroscopy, activating the Andreev (particle-to-hole) scattering. Another empirical method to probe these bound states is feasible in the Josephson junctions (Fig. 2.4), where a narrow normal region is sandwiched between two external superconducting leads. As a side remark concerning the analytical details discussed by B. Tüzemen in subsection 2.4, I must object against his estimation of the equal particle and hole spectral weights presented in equation (2.85). For non-zero energy E these spectral weights are not identical, $|u_2|^2 \neq |v_2|^2$, as can be inferred from Eqn. (2.84). Only at the Fermi energy, $E = 0$, the in-gap Andreev states comprise the same spectral weights of the particle and hole ingredients.

Chapter 3 describes methodological details useful for investigating the ferron-type objects. To accomplish it the author applied the density functional approach. He explained its conventional formulation, based on the Hohenberg-Kohn theorems, and discussed its extension to the superfluid state, where the complex pairing parameter (3.10) must be selfconsistently determined. Specific equations of such superfluid local density approximation (SLDA) were presented for the unpolarized and spin-imbalanced cases, respectively. B. Tüzemen addressed also the regularization issue arising from divergence of the kinetic and anomalous densities. To overcome this problem he imposed the ultraviolet cutoff E_c in energy integrals, rescaling the interaction strength $g(\mathbf{r}) \rightarrow g_{eff}(\mathbf{r})$. Further methodological improvement was needed in the studies of non-stationary superfluid systems. To capture their dynamics the author described time-dependent versions of SLDA and Bogoliubov de Gennes (3.26) techniques. He pointed out computational difficulties encountered in numerical algorithm for solving a huge number ($\sim 10^5$) of the coupled differential equations and discussed the employed methods.

Chapter 4 presents the original results obtained by the author for finite-size magnetic region created over certain time-interval inside the superfluid system near its unitary limit. For specific computations he imposed the Gaussian shape of this ferromagnetic region, formally represented by the spin-resolved potential (4.1) which locally trapped $\downarrow$ fermions and depleted $\uparrow$ ones, respectively. Such magnetic potential was turned on continuously over a short time interval ($25\varepsilon_F$), existed through a duration time ($100\varepsilon_F$), and was subsequently switched off smoothly. This local (in time and space) magnetic deformation substantially affected the complex order parameter in a way, resembling superconductor - ferromagnet - superconductor junction. Using the time-dependent asymmetric SLDA approach B. Tüzemen examined the resulting ferron of three-dimensional system. Ferron turned out to be pretty robust against nearly all parameters of the Gaussian potential, except some critical broadening below which it disappeared in the steady limit. The author indicated the universal feature of ferron-structures, manifested by the phase-reversal (π-shift) of the pairing field. He investigated the spatio-temporal profiles of the order parameter (magnitude and phase), inspecting emergence of its nodal regions. To get an insight into the ferron stability, he next focused on one-dimensional system, applying the Bogoliubov de Gennes method. Numerical calculations revealed an interesting relationship between the nodal regions. The author noticed a tendency towards formation of the spherical ferrons (elliptic configurations proved to be more energetically expensive). In analogy to the Josephson (S-F-S) junctions he admitted possibility of the multiple π-shifts, giving rise to formation of concentric ferrons. He also inspected various scenarios of colliding ferrons (figure 4.11) and conjectured, that ferrons would not be restricted solely to the strong pairing regime (unitary limit) but they would exist in the BCS region as well. I fully agree with this opinion.

In chapter 5 the author presented analytical and numerical results concerning the bound states formed inside the nodal parts of ferrons. These states are confined to a concentric region of a width comparable to the coherence length, where the pairing field is strongly suppressed and its phase is shifted by π (figure 5.1). Considering a piece of such normal-like region in 2-dimensional system, B. Tüzemen determined the quantized energies (5.11) and inspected these bound states against spin-imbalance δN. In my opinion, to some extent such Andreev quasiparticles are reminiscent of the Caroli - de Gennes - Matricon states appearing at energies $E_n = (n + 1/2)\Delta^2/\varepsilon_F$ in a vortex of type-II superconductor [Phys. Lett. **9**, 307 (1964)]. Since the normal-like (nodal) region exists here between the phase-differing superconducting parts, obviously it was more realistic to model it as a planar S-F-S junction (Eqn. 5.14). The author investigated the Andreev bound states of small and large ferrons, paying attention to their angular momenta (figure 5.5). He also considered the role of thermal effects (subsection 5.4), noticing that at certain temperature (sensitive to δN) the nodal region might fade away, destroying the Andreev bound states.

Chapter 6 is devoted to investigation of the dynamic properties of ferrons, which were dragged over certain time-interval with a velocity v_{drag} and afterwards the external driving potential was switched off. The author observed, that ferrons kept moving, but with much smaller final velocity v_{final}. In general, the final drift v_{final} was proportional to the driving velocity v_{drag}. This observation, however, terminated at certain critical velocity which proved to be detrimental to ferrons. B. Tüzemen determined its value with respect to the spin imbalance δN, the pairing strength $|\Delta|$ and other parameters. Furthermore, he provided analytical reasoning for the critical velocity from the hydrodynamic treatment. He also addressed a fragile stability of ferron placed in a neighborhood of the single vortex (figure 6.5). The last part of chapter 6 discussed estimations of the effective ferron mass. The author identified two contributions, one related to the ferron size and another one due to its superfluid environment. The latter proved to be nearly irrelevant for large enough ferrons and/or sufficiently strong pairing potentials.

The doctoral thesis is clearly written and well organized. Each chapter provides the self-contained information about specific properties of ferrons, summarized again in the last chapter along with a perspective for future studies. Let me comment on certain questionable points of this thesis. When studying stability of the ferron structures and analyzing the mechanism responsible for appearance of the nodal region the author refered several times to '*excited phonons*'. I am afraid, that any influence of phonons is not justified because all numerical and analytical computations are performed within the microscopic scenario, assuming two-body attraction between fermions. Such pairing potential does not have to be induced by phonons. Moreover, in the ultracold atom superfluids (often emphasized by the authors) it would solely originate from the Feshbach scattering. For this reason, literally speaking, phonons can hardly be responsible for any effects presented in this thesis. Certainly it would be interesting (but computationally quite challenging) to consider the influence of phonons, because they might affect the dynamics of ferrons. My other remark (or rather suggestion) refers to the Andreev bound states formed in the subgap energy window. I think it would be worthwhile to inspect spatial profiles and line-broadenings (life-time) of these nodal region quasiparticles in the stationary limit. All necessary details can be inferred from the Bogoliubov de Gennes approach and maybe also from semiclassical considerations [Rev. Mod. Phys. **78**, 373 (2006)].

I have only a few minor remarks to the editorial side. In section 2.3 the Bogoliubov transformation (2.39) should consist of the particle and hole contributions denoted by the same momentum $\mathbf{k}$, instead of $\mathbf{k}'$. In page 24 '*the pairing field suppose to vanish*' should probably read '*the pairing field is supposed to vanish*'. In page 46 the meaning of '*Lapcaian*' is unclear. In page 50 the phrase '*We, typically, set* $t_{on} = t_{off} = 25t\epsilon_F$' is inconsistent with the profile of the red line displayed in right h.s. panel of figure 4.1. In page 62 there is a grammar mistake '*does not induces*'. Similar (singular-plural) mistakes appear in page 72 '*figures shows*' and in page 89 '*energies* $E_\pm$ *undergoes*'. These typos are rather meaningless, therefore I don't change my very positive opinion about high quality of the thesis.

Summarizing, Buğra Tüzemen performed the systematic and valuable analysis of ferron-type structures intentionally created in the fermion superconducting/superfluid hosts. Using two different techniques, based on: (i) time-dependent formulation of the Bogoliubov de Gennes equations and (ii) superfluid extension of the local density approximation he obtained a number of original results characterizing the static and dynamic properties of ferrons. Their unique character, arising from synergy of magnetism and pairing mechanism, could be verified experimentally in the ultracold fermion-atom superfluid systems. I believe that signatures of ferrons can be also realized on surfaces of conventional superconductors decorated with nanoscopic magnetic islands, which could be easily probed by the scanning spectroscopy. Such nanostructures are currently of great interest, because by properly shaping their magnetic texture one can induce the topologically nontrivial superconducting phase hosting the protected boundary modes. Thus a variety of promising perspectives for applying and extending the ideas discussed in this thesis is possible, both in the ultracold atom and solid state realizations of quantum superfluids. I my opinion, **Buğra Tüzemen satisfies all formal and customary criteria, qualifying him to obtain the doctoral degree in the discipline of physics**.

Tadeusz Domański
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